D–8446

Sub. Code
31111

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION, MAY 2025.

First Semester

ALGEBRA — I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours | Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define a group of in G.

2. Prove that $a-b$ is an even integers in an equivalence relation $a \sim b$, where $a,b \in S$.

3. Define homomorphism of a group.

4. Define internal direct product of $N_1, N_2, ..., N_n$.

5. Define commutative ring.

6. Define solvable of a group.

7. Define the imbedded ring.

8. Define Euclidean ring.

9. Describe the integer monic.

10. Define the maximal ideal.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If A, B, C are sets the prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.

Or

(b) State and prove Euler's theorem.

12. (a) If $O(G) = P^2$ where P is prime, then prove that G is abelian.

Or

(b) Prove that $N(a)$ is a subgroup of G.

13. (a) If G and G' are isomorphic abelian group, then prove that for every integers s, $G(S)$ and $G'(S)$ are isomorphic.

Or

(b) Prove that any field is an integral domain.

14. (a) Let R be a commutative ring with unit element whose only ideals are (O) and R itself, then prove that R is a field.

Or

(b) If P is a prime number of the form $4n+1$, then solve the congruence, $x^2 \equiv -1 (\bmod P)$.

15. (a) If $f(x)$ and $g(x)$ are primitive polynomial then prove that $f(x) g(x)$ is a primitive polynomial.

Or

(b) If R is a unique factorization domain then prove that $R[x]$ is unique factorization domain.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. The subgroup N of G is a normal subgroup of G if and only if every left coset of N in G is a right coset of N in G.

17. State and prove Cauchy's theorem.

18. Prove that two abelian groups of order P^n are isomorphic if and only if they have the same invariants.

19. Prove that $J[i]$ is a Euclidean ring.

20. State and prove the Eisenstein criterion theorem.

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D–8447

Sub. Code
31112

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION, MAY 2025.

First Semester

ANALYSIS — I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define bounded above set.

2. Prove that $i^2 = -1$.

3. Define closure.

4. What do you mean by Cauchy sequence?

5. What do you mean by connected set?

6. Write a short notes on power series.

7. Define accumulation point.

8. What do you mean by interior point?

9. Write the statement of mean-value theorem.

10. Define compact set.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that every infinite subset of a countable set is countable.

Or

(b) Prove that a set E is open if and only if its complement is closed.

12. (a) If P is a limit point of a set E, then prove that every neighbourhood of P contains infinitely many points of E.

Or

(b) Suppose $\{s_n\}$ is monotonic. Prove that $\{s_n\}$ converges if and only if it is bounded.

13. (a) Prove that the subsequential limits of a sequence $\{P_n\}$ in a metric space X form a closed subset of X.

Or

(b) Prove that the series $\Sigma \frac{1}{n^p}$ converges if $P > 1$ and diverges if $P \leq 1$.

14. (a) Prove that a mapping f of a metric space X into a metric space Y is continuous on X if and only if $f^{-1}(V)$ is open in X for every open set V in Y.

Or

(b) If x is an accumulation point of S, then prove that every n-ball $B(x)$ contains infinitely many points of S.

15. (a) State and prove cantor intersection theorem.

Or

(b) State and prove Generalized mean-value theorem.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. State and prove Bolzano - Weierstrass theorem.

17. State and prove Heine - Borel theorem.

18. State and prove Chain rule for derivatives.

19. State and prove Taylor's formula with Remainder.

20. State and prove implicit function theorem.

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D–8448

Sub. Code
31113

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2025.

First Semester

ORDINARY DIFFERENTIAL EQUATIONS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Find all solutions of $y'' + 16y = 0$.

2. State the uniqueness theorem of linear equation with constant coefficients.

3. Determine whether the functions $\phi_1(x) = x^2$ and $\phi_2(x) = 5x^2$, $x \in (-\infty, \infty)$ are linearly dependent or independent.

4. Find the Wronskian of the set $\left(e^x, \cos x, \sin x\right)$.

5. Verify that $\phi_1(x) = e^x$, $x > 0$ is a solution of $xy'' - (x+1)y' + y = 0$.

6. State existence theorem for analytic coefficients.

7. Show that $P_n(-x) = (-1)^n P_n(x)$.

8. Compute the indicial polynomial of $x^2 y'' + xy' + \left(x^2 - \frac{1}{4}\right) y = 0$.

9. Write the Bessel's equation of order α and state its solution.

10. Determine whether the equation $e^x\,dx + e^y\,(y+1)dy = 0$ is exact.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that two solutions ϕ_1, ϕ_2 of $L(y)=0$ are linearly independent on an internal I if and only if $W(\phi_1, \phi_2)(x) \neq 0$.

Or

(b) Find all solutions of the equation $y'' = y' - 2y = e^{-x}$.

12. (a) Consider the equation $y''' - 4y' = 0$.

(i) Compute three linearly independent solutions

(ii) Compute the Wronskian of the solutions found in(i)

(iii) Find that solution ϕ satisfying $\phi(0)=0, \phi'(0)=1, \phi''(0)=0$.

Or

(b) Find the solution ψ of the equation $y''' + y'' + y' + y = 0$ which satisfies $\psi(0)=0, \psi'(0)=1$, $\psi''(0)=0$.

13. (a) One solution of $x^3 y''' - 3x^2 y'' + bx\,y' - 6y = 0$ for $x > 0$ is $\phi_1(x) = x$. Find a basis for the solutions for $x > 0$.

Or

(b) With the usual notations, prove that $\int_{-1}^{1} P_n^2(x)\,dx = \frac{2}{2n+1}$.

14. (a) Find a basis for the solution of the second order Euler equation $x^2 y'' + xy' + y = 0$, for $x \neq 0$ on an internal.

Or

(b) Write down the Laguerre equation. Also prove that Laguerre equation has a regular singular point at $x = 0$.

15. (a) Show that the function $f(x, y) = x^2 |y|$ satisfies a Lipschitz condition on $R: |x| \leq 1$, $|y| \leq 1$. Also prove that $\frac{\partial f}{\partial y}$ does not exist at $(x, 0)$ if $x \neq 0$.

Or

(b) Consider the initial value problem $y'_1 = y_2^2 + 1$, $y'_2 = y_1^2$ $y_1(0) = 0$, $y_2(0) = 0$. If this problem is denoted by $y' = f(x, y)$, $y(0) = y_0$. What are f and y_0? Also compute the first three successive approximations ϕ_0, ϕ_1, ϕ_2.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Let ϕ be any solution of $L(y)=y''+a_1y'+a_2y=0$ on an interval I containing a point x_0. Prove that for all x in I,

$$\|\phi(x_0)\| e^{-k|x-x_0|} \leqq \|\phi(x)\| \leqq \|\phi(x_0)\| e^{k|x-x_0|} \text{ where}$$

$$\|\phi(x)\| = \left[|\phi(x)|^2 + |\phi'(x)|^2\right]^{1/2}, \; k=1+|a_1|+|a_2|.$$

17. Find two linearly independent power series solutions of the equation $y''-xy'+y=0$

18. (a) Show that $x^{1/2} J_{1/2}(x) = \dfrac{\sqrt{2}}{\Gamma(1/2)} \sin x$.

(b) Define the Gamma function. Prove that following

(i) $\Gamma(z+1)=z\Gamma(z)$;

(ii) $\Gamma(n+1)=n!$;

(iii) $\Gamma(1)=1$.

19. Let M, N be two real-valued functions which have continuous first partial derivatives on some rectangle $R: |x-x_0| \leqq a$, $|y-y_0| \leqq b$. Prove that the equation $M(x, y)+N(x, y)\, y'=0$ is exact in R if and only if $\dfrac{\partial m}{\partial y} = \dfrac{\partial N}{\partial x}$ in R.

20. Consider the initial value problem $y'=3y+1, y(0)=2$.

(a) Show that all the successive approximations $\phi_0, \phi_1, \ldots$ exist for all real x.

(b) Compute the first four approximations $\phi_0, \phi_1, \phi_2, \phi_3$ to the solution

(c) Compute the solution by actual method.

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D–8449

Sub. Code
31114

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION, MAY 2025.

First Semester

TOPOLOGY — I

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Prove that a finite product of countable sets in countable.

2. Define finite complement topology.

3. Define subspace topology.

4. State the sequence lemma.

5. Prove that the image of a connected space under a continuous map is connected.

6. Define locally connected at x.

7. Prove that every closed subspace of a compact space is compact.

8. Define sequentially compact.

9. Define regular space.

10. Define completely regular.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that the topologies of $\mathbb{R}_l$ and $\mathbb{R}_k$ are strictly finer than the standard topology on $\mathbb{R}$, but are not comparable with one another.

Or

(b) Prove that the collection

$$S = \left\{ \pi_1^{-1}(U) \big/ U \text{ open in } X \right\} \cup \left\{ \pi_2^{-1}(V) \big/ V \text{ open in } Y \right\}$$ is a sub basis for the product topology on $X \times Y$.

12. (a) State and prove Maps into products theorem.

Or

(b) State and prove the uniform limit theorem.

13. (a) State and prove intermediate value theorem.

Or

(b) Prove that a space X is locally connected if and only if for every open set U of X, each component of U is open in X.

14. (a) Prove that every compact subspace of a Hausdorff space is closed.

Or

(b) State and prove the Lebesgue number lemma.

15. (a) Let X be a topological space. Let one-point set in X be closed. Prove that X is regular if and only if given a point x of X and a neighborhood U of x, there is a neighborhood V of x such that $\overline{V} \subset U$.

Or

(b) Prove that every compact Hausdorff space is normal.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. State and prove Rules for constructing continuous function theorem.

17. Prove that the topologies on $\mathbb{R}^n$ induced by the Euclidean metric d and the square metric ρ are the same as the product topology on $\mathbb{R}^n$.

18. State and prove the tube lemma.

19. Let X be a pace. Then prove that X is locally compact Hausdorff if and only if there exists a space Y satisfying the following conditions :

(a) X is a subspace of Y.

(b) The set $Y - X$ consists of a single point

(c) Y is a compact Hausdorff space

If Y and Y' are two spaces satisfying these conditions, then prove that there is a homeomorphism of Y with Y' that equals the identity map on X.

20. State and prove Urysohn lemma.

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D–8450

Sub. Code
31121

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION, MAY 2025.

Second Semester

ALGEBRA — II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours — Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define homomorphism.

2. Define linear span.

3. Define root of polynomial $P(x)$ in $F[x]$.

4. Define inner product space.

5. Define characteristic root.

6. Define Galois group.

7. Define basic Jordan block.

8. Define symmetric and skew symmetric matrix.

9. Define normal transformation.

10. If $T \in A(V)$ is unitary, then prove that $TT^* = 1$.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) If V is the internal direct sum of $U_1,...,U_n$, then prove that V is isomorphic to the external direct sum of $U_1,...,U_n$.

Or

(b) Prove that $L(S)$ is a subspace of a vector space V over the field F.

12. (a) If V is finite-dimensional and $v_1 \neq v_2$ are in V, prove that there is an $f \in \hat{V}$ such that $f(v_1) \neq f(v_2)$.

Or

(b) Prove that if a,b in K are algebraic over F then $a \pm b$, ab and a/b (if $b \neq 0$) are all algebraic over F. In other words, the elements in K which are algebraic over F from a subfield of K.

13. (a) Prove that any finite extension of a field of characteristics 0 is a simple extension.

Or

(b) Prove directly that any automorphism of K must leave every rational number fixed.

14. (a) Prove that two nilpotent linear transformations are similar if and only if they have the same invariants.

Or

(b) If A is invertible, prove that $(A^{-1})' = (A')^{-1}$.

15. (a) Prove that the linear transformation T on V is unitary if and only if it takes an orthonormal basis of V into an orthonormal basis of V.

Or

(b) If N is normal and $AN = NA$, prove that $AN^* = N^*A$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. If V and W are of dimensions m and n, respectively, over F then prove that $Hom(V, W)$ is of dimension mn over F.

17. If L is a finite extension of K and if K is a finite extension of F, then prove that L is a finite extension of F. Moreover, $[L:F] = [L:K]:[K:F]$.

18. Let K be the splitting field of $f(x)$ in $F[x]$ and let $P(x)$ be an irreducible factor of $f(x)$ in $F(x)$. If the roots of $P(x)$ are $\alpha_1, \ldots, \alpha_r$ then prove that for each i there exists an automorphism σ_i in $G(K, F)$ such that $\sigma_i(\alpha_i) = \alpha_i$.

19. Prove that there exists a subspace W of V, invariant under T, such that $V = V_1 \oplus W$.

20. Let G be a finite abelian group enjoying the property that the relation $x^n = e$ is satisfied by at most n elements of G, for every integer n. Then prove that G is a cyclic group.

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D–8451

Sub. Code
31122

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION, MAY 2025.

Second Semester

ANALYSIS — II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define Stieltjes integral.

2. Define unit step function.

3. If $f(x) = 0$ for all irrational x, $f(x) = 1$ for all rational x, prove that $f \notin R$ on $[a,b]$ for every $a < b$.

4. Define equicontinuous function.

5. Let $f_n(x) = \dfrac{x^2}{x^2 + (1 - nx)^2}$, $0 \le x \le 1$, $n = 1,2,3,...$ show that no subsequence of $\{f_n\}$ converge uniformly.

6. Define gamma function.

7. Define measurable set.

8. Define Borel set.

9. Define integrable function.

10. Define orthonormal set.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that $\int_{-a}^{b} f d\alpha \leq \int_{a}^{-b} f d\alpha$.

Or

(b) If $a < s < b$, f is bounded on $[a,b]$, f is continuous at s $\alpha(x) = I(x-s)$ then prove that $\int_{a}^{b} f d\alpha = f(s)$.

12. (a) If $f \in \Re(\alpha)$ on $[a,b]$ and if $|f(x)| \leq M$ on $[a,b]$, then prove that $\left| \int_{a}^{b} f\, d\alpha \leq M[\alpha(b) - \alpha(a)] \right|$.

Or

(b) Let $f_n(x) = n^2 x(1-x^2)^n$, $0 \leq x \leq 1$, $n = 1,2,....$ show that $\int_{0}^{1} f_n(x)dx = \frac{n^2}{2n+2} \to +\infty$ as $n \to \infty$.

13. (a) Given a double sequence $\{a_{ij}\}$, $i = 1,2,....; j = 1,2,....,$ suppose that $\sum_{j=1}^{\infty} |a_{ij}| = b_j$, $i = 1,2,....$ and Σb_i converges. Prove that $\sum_{i=1}^{\infty} \sum_{j=1}^{\infty} a_{ij} = \sum_{j=1}^{\infty} \sum_{i=1}^{\infty} a_{ij}$.

Or

(b) Show that $2 \int_{0}^{\pi/2} (\sin\theta)^{2x-1} (\cos\theta)^{2y-1} d\theta = \frac{\left\lceil(x) \ \left\lceil(y)\right.\right.}{\sqrt{(x+y}}$.

14. (a) For every $A \in \zeta$, prove that $\mu^*(A) = \mu(A)$.

Or

(b) Let f be a differentiable function on $[a,b]$. Show that f' is measurable on $[a,b]$.

15. (a) State and prove Fatou's theorem.

Or

(b) If $f \in L^2(\mu)$ and $g \in L^2(\mu)$, then prove that $f+g \in L^2(\mu)$ and $\|f+g\| \leq \|f\| + \|g\|$.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. If γ' is continuous on $[a,b]$, then prove that γ is rectifiable and $\Lambda(\gamma) = \int_a^b |\gamma'(t)| dt$.

17. Prove that there exists a real continuous function on the real line which is nowhere differentiable.

18. State and prove Striling's formula.

19. State and prove Lusin's theorem.

20. State and prove Lebesgue's dominated convergence theorem.

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D–8452

Sub. Code
31123

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION, MAY 2025.

Second Semester

TOPOLOGY — II

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours　　　　　　　　Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define normal space.

2. Define completely regular space.

3. Define G_δ – set.

4. Define open refinement.

5. Define paracompact space.

6. What is meant by topologically complete space?

7. Define equicontinuous function.

8. Is the subspace [–1, 1] of R, both complete and totally bounded? Justify.

9. Define topological dimension.

10. Show that the cantor set has dimension O.

PART B — $(5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that if X is completely regular, the X can be imbedded in $[0, 1]^J$ for some J.

Or

(b) Prove that a subspace of a completely regular space is completely regular.

12. (a) Let a be a locally finite collection of subsets of X. Prove that $\overline{\bigcup_{A \in a} A} = \bigcup_{A \in a} \overline{A}$.

Or

(b) Let X be a metrizable space. Prove that X has a basis that is countably locally finite.

13. (a) Prove that every metrizable space is paracompact.

Or

(b) Prove that a metric space X is complete if every Cauchy sequence in X has a convergent subsequence.

14. (a) Show that if X and Y are locally compact Hausdorff, then composition of maps $\mathcal{C}(X,Y) \times \mathcal{C}(Y,Z) \to \mathcal{C}(X,Z)$ is continuous.

Or

(b) Define empty interior and a Baire space. Give an example for each one.

15. (a) Let X be a compact metric space. Then prove that $\dim X \leq m$ if and only if for every $\in > 0$ there is a finite open covering $\mathcal{B}$ of X by sets of diameter less than $\in$ such that $\mathcal{B}$ has order at most $m+1$.

Or

(b) Prove that every open subset of a Baire space is a Baire space.

PART C — $(3 \times 10 = 30$ marks)

Answer any THREE questions.

16. State and prove that Tychonoff theorem.

17. Prove that a space X is metrizable if and only if it is paracompact and locally metrizable.

18. State and prove Ascoli's theorem.

19. Let X be a space and let (Y, d) be a metric space. For the space $\mathcal{C}(X, Y)$, Prove that the compact open topology and the topology of compact convergence coincide.

20. Let $X = Y \cup Z$ where Y and Z are closed in X having finite topological dimension, prove that $\dim X = \max\{\dim Y, \dim Z\}$.

D–8453

Sub. Code
31124

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION, MAY 2025.

Second Semester

PARTIAL DIFFERENTIAL EQUATIONS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours   Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Write the Pfaffian differential equation.

2. Define direction cosines and direction ratios.

3. Define order and degree of a partial differential equation.

4. Eliminate a and b from $2z = (ax + y)^2 + b$.

5. Define general integral.

6. Define singular integral.

7. State Laplace's equation.

8. Write the one-dimensional diffusion equation.

9. Write the interior Neumann problem.

10. State the exterior Dirichlet problem.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the integral curves of the equations $\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}$.

Or

(b) Find the integral curves of the sets of equations $\frac{dx}{x+z} = \frac{dy}{y} = \frac{dz}{z+y^2}$.

12. (a) Prove that a Pfaffian differential equation in two variables always possesses an integrating factor.

Or

(b) Solve the equation $(x^2 z - y^3)dx + 3xy^2 dy + x^3 dz = 0$ first showing that it is integrable.

13. (a) Eliminate the arbitrary function f from the equations: $z = f\left(\frac{xy}{z}\right)$.

Or

(b) Find the general solution of the differential equation $x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x+y)z$.

14. (a) If $u = f(x+iy) + g(x-iy)$, where the functions f and g are arbitrary, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

Or

(b) Find a particular integral of the equation $(D^2 - D')z = 2y - x^2$.

15. (a) Find the potential function $\psi(p,z)$ in the region $0 \le p \le 1, z \ge 0$ satisfying the conditions.

(i) $\psi \to 0$ as $z \to \infty$

(ii) $\psi = 0$ on $p = 1$

(iii) $\psi = f(p)$ on $z = 0$ for $0 \le p \le 1$.

Or

(b) Derive the d'Alembert's solution of one dimensional wave equation.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that a necessary and sufficient condition that there exists between two functions $u(x,y)$ and $v(x,y)$ a relation $F(u,v) = 0$, net involving x or y explicitly is that $\frac{\partial(u,v)}{\partial(x,y)} = 0$.

17. Verify that the equation

$z(z+y^2)dx + z(z+x^2)dy - xy(x+y)dz = 0$ is integrable and find its primitive.

18. Find the solution of the equation $z = \frac{1}{2}(p^2+q^2)+(p-x)(q-y)$.

19. By separating the variables, show that the one-dimensional wave equation $\frac{\partial^z z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 z}{\partial t^2}$ has solution of the form $A\exp(\pm inx \pm inct)$, where A and n are constants.

20. If the string is released from rest in the position $Y = \frac{4_{\varepsilon}}{l^2} x(l - x)$, show that its motion is described by the equation $y = \frac{32_{\varepsilon}}{3} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^3} \sin \frac{(2n+1)\pi x}{l} \cos \frac{(2n+1)\pi ct}{l}$.

D–8454

Sub. Code
31131

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION, MAY 2025.

Third Semester

DIFFERENTIAL GEOMETRY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define the following terms :
   (a) Osculating plane;
   (b) Curvature.

2. What is meant by an involutes and evolutes?

3. State the fundamental existence theorem for space curves.

4. Find the metric for the paraboloid $\bar{r} = (u, v, u^2 - v^2)$.

5. Define a geodesic.

6. What is meant by geodesic parallels?

7. Define the geodesic curvature vector.

8. State the Meusnier's theorem.

9. Write a short notes on the Dupin indicatrix.

10. Define a developable surface.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Show that $\left[\dot{\vec{r}}, \ddot{\vec{r}}, \dddot{\vec{r}}\right] = 0$ is a necessary and sufficient condition that the curves be plane.

Or

(b) Show that the torsion of an involute of a curve is equal to $p(\sigma p' - \sigma' p)/(p^2 + \sigma^2)(c - s)$.

12. (a) Show that the necessary and sufficient condition for a curve to be a helix is the its curvature and torsion are in a constant ratio.

Or

(b) Find the coefficients of the direction which makes an angle $\frac{\pi}{2}$ with the direction whose coefficients are (l, m).

13. (a) Prove that every helix on a cylinder is a geodesic.

Or

(b) If (λ, μ) is the geodesic curvature vector, then prove that $K_g = \frac{-H\lambda}{Fu' + Gv'} = \frac{H\mu}{Eu' + Fv'}$.

14. (a) Derive the second fundamental form a surface.

Or

(b) State and establish the Euler's theorem.

15. (a) Define the edge of regression. Also prove that the tangents to the edge of regression are the characteristic lines of the developable.

Or

(b) Prove that the edge of regression of the rectifying developable has equation $\vec{R} = \vec{r} + K \dfrac{(\tau \vec{t} + k\vec{b})}{K'\tau - K\tau'}$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Obtain the curvature and torsion of the curve of intersection of the two quadric surfaces $ax^2 + by^2 + cz^2 = 1$, $a'x^2 + b'y^2 + c'z^2 = 1$..

17. If θ is the angle at the point (u,v) between the two directions given by $Pdu^2 + 2Qdudv + Rdv^2 = 0$, then prove that $\tan\theta = \dfrac{2H(Q^2 - PR)^{1/2}}{ER - 2FQ + GP}$.

18. State and establish the Liouville's formula for Kg.

19. Derive the Rodrigue's formula.

20. State and prove the Monge's theorem.

D–8455

Sub. Code
31132

DISTANCE EDUCATION

M.Sc. (Mathematics) DEGREE EXAMINATION, MAY 2025.

Third Semester

OPTIMIZATION TECHNIQUES

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define Network with an example.

2. What is meant by enumeration of cuts?

3. Define free float.

4. Write down the Red-Flagging rule.

5. Show that the set $C = \{(x_1, x_2) / x_1 \le 2, x_2 \le 3, x_1 \ge 0, x_2 \ge 0\}$ is convex.

6. Define slack variable with an example.

7. Find the value of the game for the pay off matrix.

	B_1	B_2	B_3	B_4
A_1	8	–2	9	–3
A_2	6	5	6	8
A_3	–2	4	–9	5

8. What is meant by sensitivity analysis?

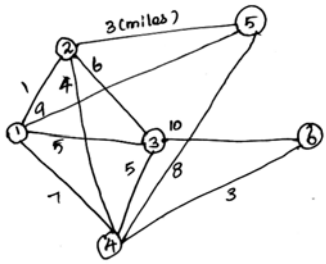
9. Write down the KKT necessary conditions for maximization problem.

10. Define steepest ascent method.

PART B — $(5 \times 5 = 25$ marks)

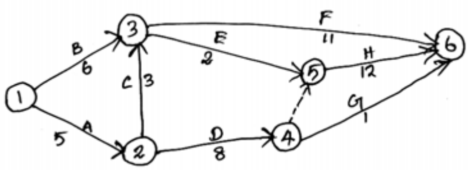
Answer ALL questions, choosing either (a) or (b).

11. (a) Midwest TV cabel company is in the process of providing cable service to 5 new housing development areas. The given figure depicts possible TV linkages among the 5 areas. The cable miles are shown on each are. Determine the most economical cable network.



Or

(b) Determine the critical path for the project network in the given figure. All the duration are in days.



12. (a) Determine and classify (as feasible and infeasible) all the basic solutions of the following system of equations.

$$\begin{pmatrix} 1 & 3 & -1 \\ 2 & -2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}.$$

Or

(b) Explain the Bounded variables Algorithm.

13. (a) Solve the games graphically. The payoff is for player A.

	B_1	B_2	B_3	B_4
A_1	2	2	3	-1
A_2	4	3	2	6

Or

(b) Write down the Dijksira's algorithm.

14. (a) Use Newton - Raphson method to solve the function $g(x) = (3x - 2)^2 (2x - 3)^2$.

Or

(b) Consider the problem

Minimize $f(x) = x_1^2 + x_2^2 + x_3^2$

Subject to

$$g_1(x) = x_1 + x_2 + 3x_2 - 2 = 0$$
$$g_2(x) = 5x_1 + 2x_2 + x_3 - 5 = 0$$

Determine the constrained extreme points.

15. (a) Find the maximum point for the function, $f(x_1, x_2, x_3) = x_1 + 2x_3 + x_2 x_3 - x_1^2 - x_2^2 - x_3^2$.

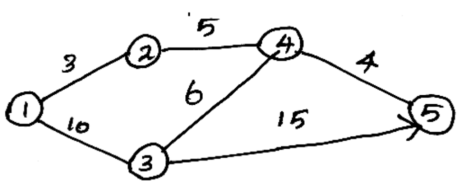
Or

(b) Write down the Dichotomous method.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. For the network in given figure, find the shortest routes between every two nodes. The distances (in miles) are given on the arcs (3, 5) is directional, so the no traffic is allowed from node 5 to node 3. All the other arcs allows two-way traffic.



17. Solve the LPS by the revised simplex method

Max $z = 5x_1 + 4x_2$

Subject to the constraint

$6x_1 + 4x_2 \leq 24$

$x_1 + 2x_2 \leq 6$

$-x_1 + x_2 \leq 1$

$x_2 \leq 2$

$and\, x_1, x_2 \geq 0.$

18. Solve the following game by linear programming

	B_1	B_2	B_3
A_1	3	–1	–3
A_2	–2	4	–1
A_3	–5	–6	2

19. Solve the LPS by separable programming method

Max $z = x_1 + x_2^4$

Subject to

$3x_1 + 2x_2^2 \leq 9$

$x_1, x_2 \geq 0.$

20. Solve the quadratic programming

Max $z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$

Subject to

$x_1 + 2x_2 \leq 2$

$x_1, x_2 \geq 0.$

D–8456

Sub. Code
31133

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION, MAY 2025.

Third Semester

ANALYTIC NUMBER THEORY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define prime number. Give an example.

2. State the Euler totient function $\phi(n)$.

3. What do you mean by Dirichlet product?

4. Define the divisor function $\sigma_{\alpha}(x)$.

5. Write down the uniqueness theorem on Bell series of an arithmetical function.

6. State the selberg identity.

7. Define Euler's constant.

8. If $a \equiv b(\bmod m)$ and if $0 \leq |b-a| < m$, then prove that $a = b$.

9. State the Wilson's theorem.

10. Find the quadratic residues modulo 11.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Given any two integers a and b, then prove that there is a common divisor d of a and b of the form $d = ax + by$, where x and y are integers? Moreover, every common divisor of a and b divides this d.

Or

(b) State and prove the Euclidean algorithm.

12. (a) With the usual notations, prove that

(i) $\phi(p^{\alpha}) = p^{\alpha} - p^{\alpha-1}$ for prime p and $\alpha \geq 1$

(ii) $\phi(mn) = \phi(m)\phi(n)$ if $(m,n) = 1$.

Or

(b) If both g and $f * g$ are multiplicate, then prove that f is also multiplicative.

13. (a) State and establish the Euler's summation formula.

Or

(b) Derive the Legendre's identity.

14. (a) Assume $(a,m) = 1$. Prove that the linear congruence $ax \equiv b(\bmod m)$ has exactly one solution.

Or

(b) State and prove the Chinese remainder theorem.

15. (a) Given integers r, d and k such that $d \mid k$, $d > 0$, $k \geq 1$ and $(r,d) = 1$. Prove that the number of elements in the set $S = \{r + td : t = 1,2,...k/d\}$ which are relatively prime to k is $\phi(k)/\phi(d)$.

Or

(b) If p is an odd positive integer, then prove that

(i) $(1 \mid p) = (-1)^{(p-1)/2}$

(ii) $(2 \mid p) = (-1)^{(p^2-1)/8}$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. For $n \geq 1$, prove the following : (a) $\phi(n) = n\prod_{p|n}\left(1 - \frac{1}{p}\right)$

(b) $\phi(n) = \sum_{d|n} \mu(d)\frac{n}{d}$.

17. (a) State and prove the generalized inversion formula.

(b) State and prove the generalized Mobius inversion formula.

18. Establish the Dirichlet's asymptotic formula.

19. State and prove the Lagrange theorem.

20. Show that the Diophantine equation $y^2 = x^3 + k$ has no solutions if K has the form $K = (4n-1)^3 - 4m^2$, where m and n are integers such that no prime $p \equiv -1 (\text{mod } 4)$ divides m.

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D–8457

Sub. Code
31134

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION, MAY 2025.

Third Semester

STOCHASTIC PROCESSES

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. What is meant by transition probability matrix?

2. Suppose that customers arrive at a counter in accordance with a Poisson process with mean rate of 2 per minute. Then the interval between any two successive arrivals follows exponential distribution with mean $1/\lambda = \frac{1}{2}$ minute. Find the probability that the interval between two successive arrivals is 4 minutes or less.

3. State first entrance theorem.

4. Write down the Fokker - Planck equation.

5. Define a Galton - Watson branching process.

6. What is meant by probability of extinction?

7. What do you mean by zero-avoiding state probabilities?

8. The arrivals at a counter in a bank occur in accordance with a Poisson process at an average rate of 8 per hour. The duration of service of a customer has exponential distribution with a mean of 6 minutes. Find the probability that an arriving customer (a) has to wait on arrival (b) 4 customers in the system.

9. Draw the state - transition - rate diagram of birth - death process.

10. Write down the Erlang's loss formula.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Suppose that the probability of a dry day (state 0) following a rainy day (state 1) is 1/3 and that the probability of a rainy day following a dry day is ½. Given that May 1 is a dry day. Find the probability that (i) May 3 is a dry day (ii) May 5 is a dry day.

Or

(b) Discuss Birth - Death process.

12. (a) If $\{N(t)\}$ is a Poisson process then prove that the auto-correlation coefficient between $N(t)$ and $N(t+s)$ is $\{t/(t+s)\}^{1/2}$.

Or

(b) Show that the mean population size of linear growth process is given by $M(t) = ie^{(\lambda-\mu)t}$.

13. (a) Explain Kolmogorov equations in Markov process.

Or

(b) Derive the distribution of the first passage time to a fixed point of wiener process.

14. (a) Show that the p.g.f. $R_n(S)$ of Y_n satisfies the recurrence relation $R_n(S) = SP(R_{n-1}(S))$, $P(S)$ being the p.g.f. of the offspring distribution.

Or

(b) Illustrate the state transition rate diagram of two-stage cyclic queueing model.

15. (a) Explain $M \mid M \mid \infty$ queueing model.

Or

(b) Derive $M \mid M(a,b) \mid 1$ queueing model.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Describe Polya's urn model.

17. Prove that under the Poisson postulates, $N(t)$ follows Poisson distribution with mean λt. That is $p_n(t) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}$, $n = 0,1,2,3...$

18. Explain Ornstein - Uhlenbeek process.

19. State and prove Yagloom's theorem.

20. Discuss M/M/S queueing model.

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D–8458

Sub. Code
31141

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION, MAY 2025.

Fourth Semester

GRAPH THEORY

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define a regular graph. Give an example.

2. What is meant by cut vertex? Give an example.

3. Write a short notes on Eulerian graph.

4. Draw 4-edge chromatic graph.

5. Define an independent set of a graph. Give an example.

6. When will you say that a graph is said to be critical?

7. State the Jordan curve theorem in the plane.

8. State the Kuratowski's theorem.

9. Draw all the graphs of tournaments on four vertices.

10. Define a network of digraph. Give an example.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that a graph is bipartite if and only if it contains no odd cycle.

Or

(b) With the usual notations, prove that $K \leq K' \leq \delta$.

12. (a) State and prove that Chvatal theorem for nonhamiltonian simple graph theorem.

Or

(b) Prove that every 3-regular graph without cut edges has a perfect matching.

13. (a) Prove that a set $S \subseteq V$ is an independent set of G if and only if $V - S$ is a covering of G.

Or

(b) With the usual notations, prove that $r(k,l) \leq \binom{k+l-2}{k-1}$.

14. (a) If G is 4-chromatic, then prove that G contains a subdivision of K_4.

Or

(b) Define dual of plane graph with an example. Also prove that $\sum_{f \in F} d(f) = 2\varepsilon$ if G is a plane graph.

15. (a) Let G be a loopless plane graph with a Hamilton cycle C. Prove that $\sum_{i=1}^{v}(i-2)(\phi'_i - \phi''_i) = 0$, where ϕ_i' and ϕ_i'' are the numbers of faces of degree i contained in $In + C$ and $Ex + C$, respectively.

Or

(b) Show that each vertex of disconnected tournament D with $v \geq 3$ is contained in a directed $k-$cycle, $3 \leq k \leq v$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove the Cayley's formula.

17. State and establish the Hall's theorem.

18. If G is simple graph, then prove that either $\psi' = \Delta$ (or) $\psi' = \Delta + 1$.

19. If G is a connected plane graph, then prove that $\gamma - \varepsilon + \phi = 2$. Also if G is a simple planar graph with $v \geq 3$, then prove that $\varepsilon \leq 3\gamma - 6$.

20. State and prove the max-flow min-cut theorem.

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D–8459

Sub. Code
31142

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION, MAY 2025.

Fourth Semester

FUNCTIONAL ANALYSIS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define a Cauchy sequence.

2. What do you mean by equivalent norm?

3. What is meant by Bounded linear operator?

4. Define a linear functional.

5. What do you mean by dual space?

6. Define a projection.

7. Write a short note on inner product space.

8. Define a Hilbert space.

9. Define reflexivity of Hilbert space.

10. What do you mean Hilbert - adjoint operator?

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Prove that on a finite dimensional vector space X, any norm $\|\cdot\|$ is equivalent to any other norm $\|\cdot\|_0$.

Or

(b) If a normed space X has the property that the closed unit ball $M = \{x / \|x\| \le 1\}$ is compact, then prove that X is finite dimensional.

12. (a) Let X, Y be vector spaces, both real or both complex. Let $T : D(T) \to Y$ be a linear operator with domain $D(T) \subset X$ and range $k(T) \subset Y$. Prove that if T^{-1} exists, it is a linear operator.

Or

(b) Let $T : D(T) \to Y$ be a linear operator, where $D(T) \subset X$ and X, Y are normed spaces. Prove that T is continuous if and only if T is bounded.

13. (a) State and prove Cauchy - Schwarz inequality.

Or

(b) Let T be a linear operator if $\dim D(T) = n < \infty$ then prove that $\dim k(T) \le n$.

14. (a) If Y is a closed subspace of a Hilbert space H, then prove that $Y = Y^{\perp\perp}$.

Or

(b) Let H_1, H_2 be Hilbert spaces, $S : H_1 \to H_2$ and $T = H_1 \to H_2$ bounded linear operator's and α be any scalar. Prove that

(i) $(S+T)^* = S^* + T^*$

(ii) $(\alpha T)^* = \overline{\alpha} T^*$.

15. (a) Let $T: H \to H$ be a bounded linear operator on a Hilbert space H. Prove that

(i) If T is self-adjoint, $\langle T_x, x \rangle$ is real for all $x \in H$

(ii) If H is complex and $\langle T_x, x \rangle$ is real for all $x \in H$ the operator T is self adjoint.

Or

(b) State and prove strong operator convergence theorem.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Prove that in a finite dimensional normed space X, any subset $M \subset X$ is compact if and only if M is closed and bounded.

17. If Y is a Banach space, then prove that $B(X, Y)$ is a Banach space.

18. State and prove Riesz's theorem on Hilbert space.

19. State and prove Baire's category theorem.

20. State and prove open mapping theorem.

D–8460

Sub. Code
31143

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION, MAY 2025.

Fourth Semester

NUMERICAL ANALYSIS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. Define order, convergence and asymptotic error constant.

2. State the Sturm theorem.

3. What is matrix norm? Give the properties.

4. Define the rate of convergence of an iterative method.

5. What do you mean by finite elements and knots?

6. Define a cubic spline.

7. Define the best approximation.

8. What is extrapolation method?

9. Define relatively stable.

10. State the Euler method.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) Find the number of real and complex roots of the polynomial equation $P_4(x) = 4x^4 + 2x^2 - 1 = 0$ using Sturm sequences.

Or

(b) Perform one iteration of the Bairstow method to extract a quadratic factor $x^2 + px + q$ from the polynomial $x^4 + x^3 + 2x^2 + x + 1 = 0$. Use the initial approximation $p_0 = 0.5, q_0 = 0.5$.

12. (a) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ using the iterative method, given that its approximate inverse is $B = \begin{bmatrix} 1.8 & -0.9 \\ -0.9 & 0.9 \end{bmatrix}$ perform two iterations.

Or

(b) For the matrix $A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 5 & 2 \\ 2 & 2 & 3 \end{bmatrix}$, (i) find all the eigen values and the corresponding eigen vectors. (ii) Verify $S^{-1}AS$ is a diagonal matrix, where S is the matrix of eigen vectors.

13. (a) Using the following values of $f(x)$ and $f'(x)$

x :	–1	0	1
$f(x)$:	1	1	3
$f'(x)$:	–5	1	7

Estimate the values of $f(-0.5)$ and $f(0.5)$ using piecewise cubic Hermit interpolation.

Or

(b) Obtain a linear polynomial approximation to the function $f(x)=x^3$ on the interval [0, 1] using the least squares approximation with $W(x)=1$.

14. (a) A differentiation rule of the form $hf'(x_2)=\alpha_0 f(x_0)+\alpha_1 f(x_1)+\alpha_2 f(x_3)+\alpha_3 f(x_4)$ where $x_j=x_0+jh$, $j=0,1,2,3,4$ is given. Determine the values of $\alpha_0,\alpha_1,\alpha_2$ and α_3 so that the rule is exact for a polynomial of degree 4 (i) find the error term (ii) obtain an expression for the round-off error in calculating $f'(x_2)$.

Or

(b) Find the Jacobian matrix for the system of equations $f_1(x,y)=x^2+y^2-x=0$, $f_2(x,y)=x^2-y^2-y=0$ at the point (1, 1).

15. (a) Solve the initial value problem $u'=-2tu^2$, $u(0)=1$ with $h=0.2$, on the interval [0, 0.4], using the backward Euler method.

Or

(b) Given the initial value problem $u'=t^2+u^2$, $u(0)=0$, determine the first three non-zero terms in the Taylor series for $u(t)$ and hence obtain the value for $u(1)$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Find all the roots of the polynomial $x^4-x^3+3x^2+x-4=0$ using the Graeffe's root squaring method.

17. Find all the eigen values of the matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ -1 & 2 & 1 \end{bmatrix}$ using the Jacobi method. Interate till the off-diagonal elements, in magnitude, are less than 0.0005.

18. Obtain the cubic spline approximation for the function defined by the data :

x :	0	1	2	3
$f(x)$:	1	2	33	244

with $M(0) = 0, M(3) = 0$. Hence, find an estimate of $f(2.5)$.

19. Derive the formulas for the first derivative of $y = f(x)$ of $O(h^2)$ using (a) forward difference approximations, (b) backward difference approximation, (c) central difference approximations. When $f(x) = \sin x$, estimate $f'(\pi/4)$ with $h = \pi/12$ using the above formulas.

20. Find the remainder of the Simpson three-eight rule $\int_{x_0}^{x_3} f(x)dx = \frac{3h}{8}[f(x_0) + 3f(x_2) + f(x_3)]$, for equally spaced points $x_i = x_0 + ih$, $i = 1,2,3$. Use this rule to approximate the value of the integral $I = \int_0^1 \frac{dx}{1+x}$. Also find a bound on the error.

D–8461

Sub. Code
31144

DISTANCE EDUCATION

M.Sc.(Mathematics) DEGREE EXAMINATION, MAY 2025.

Fourth Semester

PROBABILITY AND STATISTICS

(CBCS 2018 – 2019 Academic Year Onwards)

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL the questions.

1. If c_1 and c_2 are subsets of $\mathcal{C}$ such that $c_1 \subset c_2$, then prove that $P(c_1) \leq P(c_2)$.

2. Define the probability density function of X.

3. Write down the any two properties of a distribution function.

4. Define the covariance of X and Y.

5. If the random variable X has a Poisson distribution such that $P_r(X = 1) = P_r(X = 2)$, find $P_r(X = 4)$.

6. Define the Gamma distribution. Also find $\Gamma(1)$.

7. Let X have a p.d.f. $f(x) = \frac{1}{3}$, $x = 1, 2, 3$ and zero elsewhere. Find the p.d.f. of $Y = 2X + 1$.

8. Define the order statistic of the random sample $X_1, X_2, ...X_n$.

9. Let $\overline{X}$ be the mean of a random sample of size 5 from a normal distribution with $\mu = 0$ and $\sigma^2 = 125$. Determine c so that $P_r(\overline{X} < c) = 0.90$.

10. Define a limiting distribution.

PART B — (5 × 5 = 25 marks)

Answer ALL questions, choosing either (a) or (b).

11. (a) State and prove the Baye's theorem.

Or

(b) State and establish the Chebyshev's inequality.

12. (a) Let the joint p.d.f. of X_1 and X_2 be

$f(x_1, x_2) = x_1 + x_2, 0 < x_1 < 1,\ 0 < x_2 < 1,$

$= 0$ elsewhere. Show that the random variables X_1 and X_2 are dependent.

Or

(b) Derive the moment generating function of the chi-square distribution. Deduce its mean and variance.

13. (a) Derive the p.d.f. of t - distribution.

Or

(b) Let $Y_1 < Y_2 < Y_3 < Y_4$ denote the order statistics of a random sample of size 4 from a distribution having probability density function $f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0 & elsewhere. \end{cases}$ Find the probability density function of Y_3. Also compute $P_r\left(\frac{1}{2} < Y_3\right)$.

14. (a) If $\overline{X}$ is the mean of a random sample of size n from a normal distribution with mean μ and variance 100, find n so that $P_r(\mu - 5 < \overline{X} < \mu + 5) = 0.954$.

Or

(b) Let Y_n denote the n^{th} order statistics of a random sample $X_1, X_2, ..., X_n$ from a distribution having p.d.f. $f(x) = \frac{1}{\theta}, \quad 0 < x < \theta, \quad 0 < \theta < \infty$
$= 0, \quad elsewhere$. Find the limiting distribution of Y_n.

15. (a) Let Z_n be $\chi^2(n)$. Find the limiting distribution of the random variable $Y_n = \frac{(Z_n - n)}{\sqrt{2n}}$.

Or

(b) Let $F_n(u)$ denote the distribution function of a random variable U_n whose distribution depends upon the positive integer n. Let U_n converge in probability to the constant $c \neq 0$. Prove that the random variable U_n / c converges in probability to 1.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. (a) Let X_1 and X_2 have the joint p.d.f.

$$f(x_1, x_2) = \begin{cases} 2, & 0 < x_1 < x_2 < 1 \\ 0, & elsewhere. \end{cases}$$

Find the marginal density functions and the conditional p.d.f. of X_1 given $X_2 = x_2$.

(b) With the usual notations, show that $-1 \leq \rho \leq 1$.

17. (a) If the moment generating function of a random variable X is $M(t) = \left(\frac{1}{2} + \frac{1}{2}e^t\right)^7$, find $P_r(0 \le x \le 1)$ and $P_r(X = 5)$.

(b) Find the moment generating function of a bivariate normal distribution.

18. (a) Let X have the p.d.f. $f(x) = 1,\ 0 < x < 1$, zero elsewhere. Show that the random variable $Y = -2\ln X$ has a chi-square distribution with 2 degrees of freedom.

(b) If F has an F-distribution with parameters $r_1 = 5$ and $r_2 = 10$, find a and b so that $P_r(F \le a) = 0.05$ and $P_r(F \le b) = 0.95$ and accordingly, $P_r(a < F < b) = 0.90$.

19. Let $X_1, X_2 .... X_n$ be independent random variables having normal distribution $N(\mu_1, \sigma_1^2),\ N(\mu_2,\ \sigma_2^2), ... N(\mu_n,\ \sigma_n^2)$ respectively. Prove that $Y = k_1 X_1 + k_2 X_2 + \cdots + k_n X_n$ is $N\left(\sum_{i=1}^{n} k_i \mu_i,\ \sum_{i=1}^{n} k_i^2 \sigma_i^2\right)$, where $k_1, k_2, ... k_n$ are constants.

20. State and establish the central limit theorem.
